Integrálszámítás

Alapfüggvények integráljai

Hatványfüggvények

$$\int dx = \int 1\,dx = x + C \qquad \textit{(konstans fv.:)} \quad 1 = x^0$$

$$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad (\alpha \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \textit{(reciprok fv.:)} \quad \frac{1}{x} = x^{-1}$$

Exponenciális függvények

$$\int e^x\,dx = e^x + C \qquad \textit{(általános exponenciális fv.:)} \quad \int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (0 < a \neq 1)$$

Trigonometrikus függvények

$$\int \sin x\,dx = -\cos x + C \qquad \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \cos x\,dx = \sin x + C \qquad \int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C = -\arccos x + C$$

$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + C$$

Integrálási szabályok

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx \qquad \textit{(összeg/különbség tagonkénti integrálása)}$$

$$\int c \cdot f(x)\,dx = c \cdot \int f(x)\,dx \qquad \textit{(állandó [konstans] szorzó kiemelése)}$$

$$\int f(ax+b)\,dx = \frac{1}{a}\,F(ax+b) + C \qquad \left(\Longleftarrow \int f(x)\,dx = F(x) + C\right)$$

$$\int [f(x)]^{\alpha} \cdot f'(x)\,dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1) \qquad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int e^{f(x)} \cdot f'(x)\,dx = e^{f(x)} + C$$

Parciális integrálás: $$\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)\,dx$$

Emlékeztetők

Trigonometria

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin\alpha \cdot \cos\alpha = \frac{\sin 2\alpha}{2}$$

Hatványozás

$$a^0 = 1 \qquad a^{-3} = \frac{1}{a^3} \qquad \sqrt{a} = a^{\frac{1}{2}} \qquad \frac{1}{\sqrt[4]{a^3}} = a^{-\frac{3}{4}} \qquad \left(\frac{a}{b}\right)^{-1} = \frac{b}{a} \qquad \left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2$$

$$\sqrt[4]{\sqrt[3]{\sqrt{a^7}}} = \left(\left(\left(a^7\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} = a^{\frac{7}{24}} \qquad \sqrt[4]{a\sqrt{a}} = \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{4}} = a^{\frac{3}{8}} = \sqrt[8]{a^3}$$

Algebrai átalakítások után integrálható függvények

Mintapélda :

$$\begin{aligned}
&\int \left(\frac{x^4+2x^3-5x^2+7x-2}{x^2} + \frac{4}{\sqrt[3]{x^2}} - \sqrt{x\sqrt{x}} \right) dx = \\
&= \int \left(x^2 + 2x - 5 + \frac{7}{x} - 2x^{-2} + 4x^{-\frac{2}{3}} - x^{\frac{3}{4}} \right) dx = \\
&= \frac{x^3}{3} + x^2 - 5x + 7\ln|x| - 2\frac{x^{-1}}{-1} + 4\frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + C = \\
&= \frac{x^3}{3} + x^2 - 5x + 7\ln|x| + \frac{2}{x} + 12\sqrt[3]{x} - \frac{3}{7}\sqrt[3]{x^7} + C
\end{aligned}$$

Feladatok

$\int \frac{2+3x+4x^2}{x}\,dx =$

$\int \left(x\sqrt{x} + e^{\pi}\right) dx =$

$\int \frac{2^x}{3^x}\,dx =$

$\int \frac{3x^4}{\sqrt{x}}\,dx =$

$\int \sqrt[3]{x\sqrt[4]{x^2\sqrt[5]{x^2}}}\,dx =$

$\int (x^3-5)^2\,dx =$

$\int \frac{2^x - 3^x + 8}{7^x}\,dx =$

$\int \frac{x^2-9}{x+3}\,dx =$

$\int \sqrt{x^3} \cdot \sqrt[3]{x^2}\,dx =$

$\int \frac{2\sqrt{x} - 3x^4 + \sqrt[3]{x} - 2}{x\sqrt{x}}\,dx =$

$\int \frac{3}{4^x}\,dx =$

Szabály :

$$\int f(ax+b)\,dx = \frac{1}{a}\,F(ax+b)+C \qquad \left(\Leftarrow \int f(x)\,dx = F(x)+C\right)$$

Mintapélda :

$$\int (3x+2)^5\,dx = \frac{1}{3}\cdot\frac{(3x+2)^6}{6}+C = \frac{(3x+2)^6}{18}+C \qquad (a=3;\quad b=2;\quad f(x)=x^5)$$

Feladatok

$$\int \frac{1}{(4x-2)^2}\,dx = \qquad \int \sqrt[3]{2x-9}\,dx = \qquad \int (2008-2x)^{11}\,dx =$$

$$\int e^{x+5}\,dx = \qquad \int \frac{dx}{(x+2)^3} = \qquad \int \sqrt{6-x}\,dx =$$

$$\int \sqrt{5x+3}\,dx = \qquad \int \frac{dx}{(2x+1)^7} = \qquad \int \frac{1}{\sqrt{4x-3}}\,dx =$$

$$\int \sqrt{1-x}\,dx = \qquad \int \left(\frac{2}{3x-4}\right)^6 dx = \qquad \int \frac{6}{3x-8}\,dx =$$

$$\int \frac{1}{1+2x}\,dx = \qquad \int \lg\frac{1-x}{2}\,dx = \qquad \int \frac{dx}{\sqrt{1-x}} =$$

$$\int e^{-2x}\,dx = \qquad \int \frac{1}{x-2}\,dx = \qquad \int \sqrt[4]{1-\frac{x}{3}}\,dx =$$

$$\int \left(\frac{4x-1}{3}\right)^3 dx = \qquad \int \sqrt[4]{2+x}\,dx =$$

Szabály :

$$\int [f(x)]^{\alpha} f'(x)\, dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$$

Mintapéldák :

$$\int x \left(x^2+55\right)^7 dx = \frac{1}{2}\int \left(x^2+55\right)^7 \cdot \left(x^2+55\right)' dx = \frac{\left(x^2+55\right)^8}{16} + C \qquad (\alpha = 7; \quad f(x) = x^2+55)$$

$$\int \frac{\sqrt{\ln x}}{x}\, dx = \int (\ln x)^{\frac{1}{2}} \cdot (\ln x)'\, dx = \frac{(\ln x)^{\frac{3}{2}}}{3/2} + C = \frac{2\sqrt{\ln^3 x}}{3} + C \qquad (\alpha = \frac{1}{2}; \quad f(x) = \ln x)$$

Feladatok

$$\int x(x^2+2)^4\, dx = \qquad \int x\sqrt{1-x^2}\, dx = \qquad \int \frac{(1+\ln x)^2}{x}\, dx =$$

$$\int \frac{\ln^2 x}{x}\, dx = \qquad \int \frac{x}{\sqrt{1+x^2}}\, dx = \qquad \int \frac{1}{x\sqrt{\ln x}}\, dx =$$

$$\int e^x\sqrt{2-e^x}\, dx = \qquad \int \frac{e^x}{\sqrt{e^x-1}}\, dx = \qquad \int x^2\sqrt{(x^3-1)^5}\, dx =$$

$$\int \frac{dx}{x\sqrt{1-\ln x}} \qquad \int \frac{dx}{x\sqrt[3]{(2-\ln x)^2}} =$$

Szabály :

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

Mintapélda :

$$\int \frac{x^4}{x^5+3}\,dx = \frac{1}{5}\int \frac{(x^5+3)'}{x^5+3}\,dx = \frac{1}{5}\ln\left|x^5+3\right| + C$$

Feladatok

$$\int \frac{2x}{1+x^2}\,dx = \qquad \int \frac{e^x}{1+e^x}\,dx = \qquad \int \frac{1}{x\ln x}\,dx =$$

$$\int \frac{2x+1}{x^2+x}\,dx = \qquad \int \frac{x^2}{1-x^3}\,dx = \qquad \int \frac{x+1}{x^2+2x+2}\,dx =$$

Szabály *(Parciális integrálás)* :

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx$$

Mintapélda :

$$\begin{aligned}\int xe^x\, dx = \int x(e^x)'\, dx &= xe^x - \int (x)'e^x\, dx = \\ &= xe^x - \int e^x\, dx \\ &= xe^x - e^x + C\end{aligned}$$

Feladatok

$$\int x \ln x\, dx = \qquad \int (3x-4)e^x\, dx \qquad \int xe^{-2x}\, dx =$$

$$\int \ln x\, dx = \qquad \int \frac{\ln x}{x}\, dx = \qquad \int \log_2 x\, dx =$$

$$\int x \sin x\, dx = \qquad \int e^x \cos x\, dx = \qquad \int \operatorname{arctg} x\, dx =$$

Vegyes feladatok

$$\int \frac{e^x}{\sqrt[3]{(e^x+4)^2}}\,dx = \qquad \int xe^{-2x}\,dx = \qquad \int \frac{x}{e^x}\,dx =$$

$$\int xe^{x^2}\,dx = \qquad \int \frac{1}{x\ln^3 x}\,dx = \qquad \int 2e^{1-x}\,dx =$$

$$\int xe^{x^2}\,dx = \qquad \int xe^{x}\,dx = \qquad \int x^2e^{x}\,dx =$$

$$\int \ln(2x-3)\,dx = \qquad \int \sqrt{e^x}\,dx = \qquad \int (e^x-3)^2\,dx =$$

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = \qquad \int \lg x\,dx = \qquad \int \frac{x^4}{1+x^{10}} =$$

$$\int \frac{x}{e^{x^2}}\,dx = \qquad \int \frac{dx}{x(1+\ln x)} = \qquad \int \frac{e^{\frac{1}{x}}}{x^2} =$$

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