Vektor-Skalár függvény *(térgörbe)* $\quad \boldsymbol{R} \to \boldsymbol{R}^3$

$$\boxed{\boldsymbol{r} = \boldsymbol{r}(t) = x(t)\,\boldsymbol{i} + y(t)\,\boldsymbol{j} + z(t)\,\boldsymbol{k}}$$

ívhossz: $\quad s = \int\limits_{t_1}^{t_2} |\dot{\mathbf{r}}(t)|\,dt = \int\limits_{t_1}^{t_2} \sqrt{[\dot{x}(t)]^2 + [\dot{y}(t)]^2 + [\dot{z}(t)]^2}\,dt \qquad t \in [t_1; t_2]$

$$\left.\begin{array}{lll} \textit{érintővektor:} & \boldsymbol{t} = \boldsymbol{r}' = \dfrac{\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}|} & = \boldsymbol{n} \times \boldsymbol{b} \\ \textit{(fő)normális vektor:} & \boldsymbol{n} = \dfrac{\boldsymbol{r}''}{|\boldsymbol{r}''|} = \dfrac{\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}|} \times \dfrac{\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}|} & = \boldsymbol{b} \times \boldsymbol{t} \\ \textit{binormális vektor:} & \boldsymbol{b} = \dfrac{\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}|} & = \boldsymbol{t} \times \boldsymbol{n} \end{array}\right\} \quad \left\{\begin{array}{ll} \boldsymbol{t}, \boldsymbol{n} & \textit{simulósík} \\ \boldsymbol{t}, \boldsymbol{b} & \textit{rektifikálósík} \\ \boldsymbol{b}, \boldsymbol{n} & \textit{normálsík} \end{array}\right.$$

kísérő triéder: $\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}$ *jobbrendszert alkotó egységvektorok*

Görbület: $\quad \varkappa = |\boldsymbol{t}'| = |\boldsymbol{r}''| = \dfrac{|\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}|}{|\dot{\boldsymbol{r}}|^3} = \dfrac{1}{\varrho} \qquad$ *az érintő irányváltozásának sebessége*

Csavarodás (torzió): $\quad \tau = \boldsymbol{b}\boldsymbol{b}'\boldsymbol{t} = \dfrac{\dot{\boldsymbol{r}}\,\ddot{\boldsymbol{r}}\,\dddot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}} \times \ddot{\boldsymbol{r}}|^2} \qquad$ *a simulósík irányváltozásának sebessége*

DARBOUX-vektor: $\quad \boldsymbol{d} = \tau\boldsymbol{t} + \varkappa\boldsymbol{b}$

FRENET képletek: $\quad \left\{\begin{array}{rlr} \boldsymbol{t}' = & \varkappa\boldsymbol{n} & \\ \boldsymbol{n}' = & -\varkappa\boldsymbol{t} & +\tau\boldsymbol{b} \\ \boldsymbol{b}' = & & -\tau\boldsymbol{n} \end{array}\right\}$ $\quad$ illetve: $\quad \left\{\begin{array}{c} \boldsymbol{t}' = \boldsymbol{d} \times \boldsymbol{t} \\ \boldsymbol{n}' = \boldsymbol{d} \times \boldsymbol{n} \\ \boldsymbol{b}' = \boldsymbol{d} \times \boldsymbol{b} \end{array}\right\}$

Görbület és torzió metrikus meghatározása

$$\varkappa \;=\; 4 \cdot \lim_{P_i \to P} \frac{T(P_1P_2P_3)}{P_1P_2 \cdot P_2P_3 \cdot P_3P_3} \qquad (\leftarrow\; abc = 4rT) \qquad \textit{(K.Menger)}$$

$$\tau \;=\; \frac{9}{4} \cdot \lim_{P_i \to P} \frac{V(P_1P_2P_3P_4)}{\sqrt{T(P_1P_2P_3) \cdot T(P_2P_3P_4) \cdot T(P_3P_4P_1) \cdot T(P_1P_2P_4)}} \qquad \textit{(G.D.Darboux - Egerváry J.)}$$

(P_1, P_2, P_3, P_4 $\quad$ a görbe pontjai, melyekre: $\quad \lim P_i = P \quad i = 1; 2; 3; 4$)

Csavarvonal

$$\boldsymbol{r}(t) = R\cos t\,\boldsymbol{i} + R\sin t\,\boldsymbol{j} + at\,\boldsymbol{k} \qquad \textit{(a csavarvonal tengelye a z tengely.)}$$

$$\boldsymbol{r} = \begin{bmatrix} R\cos t \\ R\sin t \\ at \end{bmatrix} \qquad \dot{\boldsymbol{r}} = \begin{bmatrix} -R\sin t \\ R\cos t \\ a \end{bmatrix} \qquad \ddot{\boldsymbol{r}} = \begin{bmatrix} -R\cos t \\ -R\sin t \\ 0 \end{bmatrix} \qquad \dddot{\boldsymbol{r}} = \begin{bmatrix} R\sin t \\ -R\cos t \\ 0 \end{bmatrix}$$

$$\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}} = \begin{bmatrix} aR\sin t \\ -aR\cos t \\ R^2 \end{bmatrix} \qquad (\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}})\times\dot{\boldsymbol{r}} = -R(R^2+a^2)\begin{bmatrix} \cos t \\ \sin t \\ 0 \end{bmatrix} \qquad \dot{\boldsymbol{r}}\ddot{\boldsymbol{r}}\dddot{\boldsymbol{r}} = aR$$

$$|\dot{\boldsymbol{r}}| = \sqrt{R^2+a^2} \qquad |\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}| = R\sqrt{R^2+a^2}$$

érintő egységvektor $\quad \boldsymbol{t} = |\boldsymbol{r}'| = \dfrac{\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}|} = \dfrac{1}{\sqrt{R^2+a^2}}\begin{bmatrix} -R\sin t \\ R\cos t \\ a \end{bmatrix}$

(fő)normális egységvektor $\quad \boldsymbol{n} = \dfrac{(\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}})\times\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}|\,|\dot{\boldsymbol{r}}|} = \begin{bmatrix} -\cos t \\ -\sin t \\ 0 \end{bmatrix}$

binormális egységvektor $\quad \boldsymbol{b} = \dfrac{\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}|\,|\dot{\boldsymbol{r}}|} = \dfrac{1}{\sqrt{R^2+a^2}}\begin{bmatrix} a\sin t \\ -a\cos t \\ R \end{bmatrix}$

Görbület:

$$\varkappa = \frac{R}{R^2 + a^2}$$

Csavarodás, torzió:

$$\tau = \frac{a}{R^2 + a^2}$$

DARBOUX-vektor:

$$\boldsymbol{d} = \tau \boldsymbol{t} + \varkappa \boldsymbol{b} = \frac{1}{\sqrt{R^2 + a^2}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Ha adott a görbület ($\varkappa$) és a csavarodás (τ): $\quad R = \dfrac{\varkappa}{\varkappa^2 + \tau^2} \quad a = \dfrac{\tau}{\varkappa^2 + \tau^2}$

$$\boldsymbol{t} = \frac{1}{\sqrt{\varkappa^2 + \tau^2}} \begin{bmatrix} -\varkappa \sin t \\ \varkappa \cos t \\ \tau \end{bmatrix} \qquad \boldsymbol{n} = \begin{bmatrix} -\cos t \\ -\sin t \\ 0 \end{bmatrix} \qquad \boldsymbol{b} = \frac{1}{\sqrt{\varkappa^2 + \tau^2}} \begin{bmatrix} -\tau \sin t \\ -\tau \cos t \\ \varkappa \end{bmatrix}$$

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